Introduction

This discussion will review various models of drivers approaching an intersection in both through and turning lanes and what yellow change interval may be necessary in those situations. It is an extension of the work of Gazis, Herman, and Maradudin’s (GHM) in their seminal 1960 paper, “The Problem of the Amber Signal Light in Traffic Flow”. GHM proposed the basic protocol used today for setting a minimum yellow change interval for through movements that would provide a motorist with a solvable solution to the stop or go problem encountered when the yellow signal is illuminated.

Any Protocol for Setting the Minimum Yellow Interval Must Begin with a Consideration of the Stop or Go Decision Based on the Minimum Stopping Distance

A vehicle driver approaching a signalized intersection when the signal changes from green to yellow is always faced with the dilemma to either come to a complete stop or continue into and through the intersection. This is known as the “stop or go decision”. The proper decision whether to stop or go at the onset of the yellow light is entirely dependent on the driver-vehicle’s location in relation to their minimum safe and comfortable stopping distance (aka Critical Distance ($x_c$)), which is contingent on their initial approach velocity ($v_0$) at the moment the yellow illuminates, a given maximum perception-reaction time ($t_{PR}$), and a maximum deceleration ($a_{max}$).

The critical distance is not dependent on the yellow change interval duration. Rather, the yellow change interval duration is dependent on the critical distance.

For a driver responding to the onset of the yellow indication, their minimum stopping distance is comprised of the distance their vehicle travels during the time they perceive and react to the yellow indication (Perception-Reaction Distance ($x_{PR}$)) and the distance their vehicle travels when braking to a complete stop (Braking Distance ($x_{Br}$))\(^1\). Using basic kinematic equations of motion, a neutral observer can calculate the vehicle’s minimum stopping distance and hence the proper decision for the driver:

$$x_c = x_{PR} + x_{Br} = v_0 t_{PR} + \frac{v_0^2}{2a_{max}}$$ \hspace{1cm} (1)

Where:

- $x_c$ = Critical distance - the minimum safe and comfortable stopping distance (ft.)
- $v_0$ = Maximum uniform (constant) initial/approach velocity (ft./sec.)
- $t_{PR}$ = Maximum allocated driver-vehicle perception-reaction time (minimum 1.0 sec)
- $a_{max}$ = Maximum uniform (constant) safe and comfortable deceleration (maximum 10 fps\(^2\))

\(^1\)Throughout this paper, “braking distance” will exclusively refer to the distance a driver’s vehicle travels when braking from their initial velocity to a complete stop (zero velocity). The “Braking Point” will refer to the location where the driver must begin to brake to a complete stop.
\( v_0 t_{PR} \) represents the perception-reaction distance \( (x_{PR}) \) and \( \frac{v_0^2}{2a_{max}} \) represents the minimum braking distance \( (x_{BR}) \).

The intersection diagram in Figure 1 shows this concept.

At the moment the green light changes to yellow, a reasonable driver further upstream from the intersection than their minimum safe and comfortable stopping distance (position A) has sufficient distance to come to a complete stop and should to do so. For this driver, the duration of the yellow interval is irrelevant.

A driver any closer to the intersection than their minimum stopping distance (between position B and the limit line) when the yellow illuminates does not have sufficient roadway to stop before reaching the limit line. The reasonable driver must keep going and is legally entitled to do so. Therefore, the yellow indication must, at a MINIMUM, remain illuminated long enough for a reasonable driver to traverse this distance to the intersection. Any yellow interval set shorter than the time for the driver to traverse this distance to the intersection puts the driver in physical and legal jeopardy, as the red light may illuminate prior to them crossing the limit line. Set appropriately, the yellow interval will eliminate the “dilemma zone” for all driver-vehicle complexes with an assumed maximum initial approach speed, maximum perception-reaction time, and maximum safe and comfortable deceleration.

Since the minimum stopping distance is the MAXIMUM distance the driver must theoretically traverse during the yellow interval, any protocol for setting a minimum yellow interval is based on the assumption that the yellow indication illuminates at the moment the driver-vehicle reaches their minimum stopping distance (position B). We can designate this as the “worst case scenario” or “boundary condition”.

Calculating the Minimum Yellow Change Interval for Through Movements

Calculating the minimum yellow change interval \( (Y_{min}) \) is relatively straight forward for driver-vehicles that maintain their initial approach velocity while traversing the minimum stopping distance.
Since time equals distance divided by velocity \( t = \frac{x}{v} \) we can simply divide the minimum stopping distance by the vehicle’s velocity:

\[
Y_{\text{min}} = \frac{x_c}{v_0} = \frac{v_0 t_{PR} + \frac{v_0^2}{2a_{\text{max}}}}{v_0} = \frac{v_0 t_{PR}}{v_0} + \frac{\frac{v_0^2}{2a_{\text{max}}}}{v_0}
\]  

(2)

Which reduces to the well-known ITE Kinematic Formula:

\[
Y_{\text{min}} = t_{PR} + \frac{v_0}{2a_{\text{max}}}
\]  

(3)

Figure 2 graphically represents this binary stop or go condition.

A critical and undeniable concept that must be grasped is that the Kinematic Formula can only be derived if both the initial velocity \( v_0 \) which is used to calculate the minimum stopping distance and the vehicle’s velocity while traversing the minimum stopping distance is the same. Where a vehicle must slow down, such as to negotiate a turn, the initial velocity \( v_0 \) and the vehicle’s velocity while traversing the critical distance are NOT the same and the Kinematic Formula cannot be used. A new protocol must be devised for situations where a vehicle must slow down within the minimum stopping distance to negotiate a turn.
Protected and Permissive Left-Turn and Right-Turn Applications

The logic behind the methodology for determining the length of the yellow change interval is that the duration of the yellow interval should provide a “reasonable” driver who is too close to the intersection to stop safely and comfortably (i.e. closer than the minimum stopping distance) with adequate time to traverse the minimum stopping distance to the intersection and legally enter before the signal turns red. A “reasonable” driver closer to the intersection than the minimum stopping distance will proceed into the intersection when presented with a yellow indication.

Likewise, the calculation of the minimum yellow change interval for turning movements must provide the minimum time necessary for a driver who is too close to the intersection to stop safely and comfortably with adequate time to traverse the minimum stopping distance and legally enter the intersection before the onset of the red indication. Therefore, in conformance with the standard for through lane movements, any protocol for setting a minimum yellow interval for turning movements must incorporate the assumption that the yellow indication illuminates at the moment the driver-vehicle reaches their minimum stopping distance.

The calculation must also allow for the extra time necessary for a vehicle to traverse the stopping distance while decelerating from the initial approach velocity ($v_0$) to the intersection entry velocity ($v_E$) in order to safely and comfortably negotiate the turning maneuver.

In contrast to the condition where a driver is approaching a signalized intersection in a through lane, situations where a driver is approaching a signalized intersection in a turning lane are significantly more complicated. A critical additional variable is the driver’s deceleration to negotiate the turn. Where along the driver’s trajectory towards the intersection this deceleration occurs, dictates our assumptions for the driver’s initial approach velocity and minimum stopping distance.

Where Must the Driver Begin to Decelerate?

Some confusion exists with regards to where on their approach to the intersection a driver must begin to decelerate to achieve their target intersection entry speed ($v_E$). A number of possibilities exist as to where a driver can begin to decelerate. Some possibilities include:

a) before reaching the minimum stopping distance
b) within the perception-reaction distance,
c) within the braking distance.

Assuming a maximum safe and comfortable deceleration ($a_{max}$) to decelerate to a target intersection entry speed ($v_E$) greater than zero, we can determine at what location on their approach to the intersection the driver must begin to slow down. We will show that this location is within the braking distance.

Since $x = vt$, in order to calculate the distance a vehicle travels while decelerating from a higher initial velocity ($v_0$) to a lower final velocity ($v_f$), we must calculate both the vehicle’s average velocity ($v_{av}$) and the time that elapses during the deceleration from $v_0$ to $v_f$.

Average velocity ($v_{av}$) is given by:
\[ v_{av} = \frac{v_0 + v_f}{2} \]  

The time it takes to decelerate from the initial velocity to the final velocity \((t_{Dec})\) is given by:

\[ t_{Dec} = \frac{(v_0 - v_f)}{a_{max}} \]  

Therefore, the distance a vehicle travels while decelerating from a higher initial velocity \((v_0)\) to a lower final velocity \((v_f)\), (aka the deceleration distance \((x_{Dec})\)) is:

\[ x_{Dec} = v_{av} \cdot t_{Dec} = \frac{(v_0 + v_f)}{2} \cdot \frac{(v_0 - v_f)}{a_{max}} = \frac{v_0^2 - v_f^2}{2a_{max}} = x_{Ob} \]  

In a discussion regarding minimum yellow change intervals, \(v_0\) is the initial approach speed and \(v_f\) is the vehicle’s intersection entry speed \((v_E)\). The equation above can therefore be used to determine the location upstream of the limit line at which a driver MUST begin to decelerate in order to achieve their desired entry speed. This distance is their Obligatory Deceleration Distance \((x_{Ob})\).

Comparing this obligatory deceleration distance (substituting \(v_E\) for \(v_f\)):

\[ x_{Ob} = \frac{v_0^2 - v_E^2}{2a_{max}} \]  

to the driver’s braking distance (to zero velocity), which is given by the last term of Equation 1:

\[ x_{Br} = \frac{v_0^2}{2a_{max}} \]  

clearly shows that the obligatory deceleration distance is always less than the braking distance \((x_{Ob} < x_{Br})\). Therefore, a driver need not begin decelerating to their target entry speed \((v_E)\) until they are closer to the intersection than their braking distance and, by extension, their minimum stopping distance (which is the braking distance plus the perception-reaction distance).

**A Model for Driver-Vehicle Motion in Turning Lanes**

Having determined that a driver need not begin decelerating to achieve their target entry speed until they are closer to the intersection than their braking distance, we consider a reasonable model of driver-vehicle motion in turning lanes and how that model determines the minimum yellow change interval.

As stated previously, there is a range of possibilities as to where a driver might begin to decelerate on their approach to the intersection. A driver who begins their deceleration prior to reaching their minimum stopping distance (based on their initial approach speed) will have a dynamically changing minimum stopping distance along their trajectory. Depending upon when the yellow indication illuminates, such a driver may have sufficient distance to stop or may need less yellow time than a
driver that delays their deceleration until they are within their braking distance. We therefore eliminate this scenario from consideration of our vehicle motion model as it will not provide a sufficient yellow interval for some reasonable and legally acting drivers.

Likewise, we will eliminate situations where the driver begins to decelerate within the perception-reaction distance as this model of driver-vehicle motion also may require less yellow time than where a driver delays their deceleration until they are within their braking distance. Consequently, we will only consider scenarios where the driver decelerates within their braking distance. This is analogous to the model used for through movements where we base the minimum yellow interval on the “worst case scenario” or “boundary condition”.

For drivers decelerating within their braking distance, we can envision three possible models of driver-vehicle motion.

**Scenario 1**

In Scenario 1, the driver delays their deceleration until they reach their obligatory deceleration distance \(x_{Ob}\) upstream of the limit line. Upon reaching the obligatory deceleration point \(x_{Ob}\), they then decelerate at their maximum safe and comfortable deceleration \(a_{max}\) to reach their target entry velocity \(v_E\) at the moment they arrive the limit line. The intersection diagram in Figure 3 shows this concept.

![Intersection Diagram for Decision to Delay Deceleration Until Reaching the Obligatory Deceleration Point.](image)

In this model, the minimum stopping distance is divided into three distinct areas of vehicle movement, 1) the Perception-Reaction zone with a length equal to \(x_{PR}\), 2) the zone from the start of the braking distance to the obligatory deceleration point where the driver does not decelerate (a Non-Deceleration Zone with a length equal to \(x_{NDZ}\)), and 3) the Deceleration Zone with a length equal to \(x_{Dec}\) (also equal to \(x_{Ob}\)) where the driver decelerates to their target entry velocity.
Therefore, the minimum time to traverse the minimum stopping distance is the combination of 1) the
time to traverse the perception-reaction distance \( t_{PR} \), 2) plus the time to traverse the Non-
Deceleration Zone \( t_{NDZ} \), 3) plus the time to traverse the Deceleration Zone \( t_{Dec} \). By definition, this
is the minimum yellow change interval \( Y_{min} \) necessary to eliminate the dilemma zone for this model:

\[
Y_{min} = t_{PR} + t_{NDZ} + t_{Dec}
\]  

Also, recall that per the time to traverse the Deceleration Zone is given by
\[
t_{Dec} = \frac{(v_0 - v_f)}{a_{max}}
\]
as per our previous discussion (Equation (5)) and \( v_f \) is equal to the target entry velocity \( v_E \).

The time to traverse the Non-Deceleration Zone is simply the length of Non-Deceleration Zone \( x_{NDZ} \)
divided by the vehicle’s velocity. The length of the Non-Deceleration Zone is calculated by subtracting
the length of the Deceleration Zone \( x_{Dec} \) from the braking distance \( x_{BR} \).

Recall that from Equation (6), the length of the Deceleration Zone is given by:

\[
x_{Ob} = x_{Dec} = \frac{v_0^2 - v_E^2}{2a_{max}}
\]  

And that the braking distance is given by:

\[
x_{BR} = \frac{v_0^2}{2a_{max}}
\]

Therefore the length of Non-Deceleration Zone is given by:

\[
x_{NDZ} = x_{BR} - x_{Dec} = \frac{v_0^2}{2a_{max}} - \frac{v_0^2 - v_E^2}{2a_{max}} = \frac{v_0^2 - v_E^2}{2a_{max}} = \frac{v_E^2}{2a_{max}}
\]

Recall that the driver maintains their initial velocity \( (v_0) \) over this distance, so the time to traverse the
Non-Deceleration Zone is given by:

\[
T_{NDZ} = \frac{x_{NDZ}}{v_0} = \frac{v_E^2}{2a_{max}v_0}
\]

Therefore, the minimum time to traverse the minimum stopping distance (by definition the minimum
yellow change interval, \( Y_{min} \)) for this model is given by:

\[
Y_{min} = t_{PR} + t_{NDZ} + t_{Dec} = T_{pr} + \frac{v_E^2}{2a_{max}v_0} + \frac{(v_0 - v_E)}{a_{max}}
\]

\[\text{Note that the time to traverse the perception-reaction distance is, by definition, the perception-reaction time (} \text{t}_{PR} \text{).} \]

\[\text{2 Note that the time to traverse the perception-reaction distance is, by definition, the perception-reaction time (} \text{t}_{PR} \text{).} \]
Figure 4 graphically represents this model:

In Scenario 2, we conceive that the driver decelerates at a constant deceleration across the braking distance to achieve their target entry speed. The intersection diagram in Figure 5 shows this concept.

**Scenario 2**
In Scenario 2, we conceive that the driver decelerates at a constant deceleration across the braking distance to achieve their target entry speed. The intersection diagram in Figure 5 shows this concept.
In this model, the minimum stopping distance is divided into two distinct areas of vehicle movement, 1) the Perception-Reaction zone with a length equal to \( x_{PR} \), and 2) a Deceleration Zone with a length equal to the entire length of the braking distance \( x_{Br} \).

Therefore, the minimum time to traverse the minimum stopping distance is the combination of 1) the time to traverse the perception-reaction distance \( t_{PR} \), 2) plus the time to traverse the Deceleration Zone \( t_{Dec} \).

The time to traverse the Deceleration Zone \( t_{Dec} \) can be calculated by dividing the braking distance \( x_{Br} \) by the driver’s average velocity across that distance. Again, the braking distance is given by:

\[
x_{Br} = \frac{v_0^2}{2a_{max}}
\]  

and average velocity \( v_{av} \) is given by:

\[
v_{av} = \frac{v_0 + v_f}{2}
\]

Therefore the time to traverse the minimum stopping distance for this model (again substituting \( v_E \) for \( v_f \)) is given by:

\[
Y_{min} = T_{pr} + \frac{x_{Br}}{v_{av}} = T_{pr} + \frac{\frac{v_0^2}{2a_{max}}}{\frac{v_0 + v_E}{2}} = T_{pr} + \frac{v_0^2}{v_0 + v_E} = T_{pr} + \frac{v_0}{a_{max}(1 + \frac{v_E}{v_0})}
\]  

Note that although we are referencing a maximum deceleration to calculate the minimum time across the stopping distance, the driver in this model decelerates at less than their maximum safe and comfortable deceleration.

Figure 6 below graphically represents this model:
Scenario 3
In Scenario 3, we conceive that the driver begins their deceleration at the braking distance, decelerating at their maximum safe and comfortable deceleration \( (a_{\text{max}}) \) to their target entry velocity \( (v_E) \) and then traverses the remainder of the braking distance at this velocity into the intersection. The intersection diagram in Figure 7 shows this concept.

**Figure 6** - Time Graph for Decision to Decelerate with Uniform (Constant) Deceleration Across the Braking Distance to a Target Entry Velocity \( (v_E) \) greater than zero. \((Note \ a^* < a_{\text{max}})\)

**Figure 7** - Intersection Diagram for Decision to Decelerate with Maximum Deceleration Beginning at the Braking Distance to a Target Entry Velocity and then Continue at that velocity into the Intersection.
Similar to Scenario 1, in this model, the minimum stopping distance is divided into three distinct areas of vehicle movement, 1) the Perception-Reaction zone with a length equal to \( x_{PR} \), 2) a Deceleration Zone with a length equal to \( x_{Dec} \), where the driver decelerates to their target entry velocity beginning at the braking point, and 3) a Non-Deceleration “Go Zone” length equal to \( x_{Go} \) starting at the end of the Deceleration Zone where the driver continues at their target entry speed to the limit line and into the intersection.

Therefore, the minimum time to traverse the minimum stopping distance is the combination of 1) the time to traverse the perception-reaction distance \( (t_{PR}) \), 2) plus the time to traverse the Deceleration Zone \( (t_{Dec}) \) 3) plus the time to traverse the Go Zone \( (t_{Go}) \). By definition, this is the minimum yellow change interval \( (Y_{min}) \) necessary to eliminate the dilemma zone for this model.

Recall that the time to traverse the Deceleration Zone is given by:

\[
    t_{Dec} = \frac{(v_0 - v_E)}{a_{max}} \tag{5}
\]

as per our previous discussion.

The time to traverse the Non-Deceleration “Go Zone” is simply the length of Non-Deceleration “Go” Zone \( (x_{Go}) \) divided by the vehicle’s velocity across the distance. As in Scenario 1, the length of the Go Zone is calculated by subtracting the length of the Deceleration Zone from the braking distance. This distance is the same as calculated in Scenario 1 as the lengths of the Deceleration Zones are equal in both scenarios and is given by:

\[
    x_{go} = \frac{v_E^2}{2a_{max}} \tag{15}
\]

In this model, the driver’s velocity over this distance is their target entry velocity, so the time to traverse the Go Zone \( (T_{go}) \) is given by:

\[
    T_{go} = \frac{x_{go}}{v_E} = \frac{\frac{v_E^2}{2a_{max}}}{v_E} = \frac{v_E}{2a_{max}} \tag{16}
\]

Therefore, the minimum time to traverse the minimum stopping distance (by definition the minimum yellow change interval, \( Y_{min} \)) is given by:

\[
    Y_{min} = T_{pr} + t_{Dec} + T_{go} = T_{pr} + \frac{(v_0 - v_E)}{a_{max}} + \frac{v_E}{2a_{max}} \tag{17}
\]

Figure 8 below represents this model:
Which Model Provides Reasonable Drivers with a Sufficient Yellow Interval?

The logic behind the methodology for determining the length of the yellow change interval is that the duration of the yellow change interval should provide a “reasonable” driver who is too close to the intersection to stop safely and comfortably, with adequate time to traverse their minimum stopping distance to the intersection before the signal turns red. For through movements, we recognize that the “worst case scenario” or “boundary condition” that reasonable and legally behaving motorists are likely to encounter is where the yellow indication illuminates at the moment the driver-vehicle reaches their minimum stopping distance.

Likewise, for turning movements, we assume that the boundary condition is where the yellow indication illuminates at the moment the driver-vehicle reaches their minimum stopping distance. For various potential models of driver-vehicle motion in turning lanes, an additional boundary condition is the model which provides the reasonable driver with the longest time to traverse their minimum stopping distance to the intersection before the signal turns red.

We now consider each of the three scenarios discussed above to determine which model requires the longest time to traverse the minimum stopping distance. This model will then represent the boundary condition for drivers in turning lanes.

Since the reaction time is assumed to be the same in all three models, we need only examine the time to traverse the braking distance.
Note that scenarios 1 and 3 are similar in that there is a period of deceleration and a period of non-deceleration while the driver traverses the braking distance.

The time to decelerate from $v_0$ to $v_E$ is equal in both scenarios, as is the distance traveled during this period. Therefore, we need only compare the non-deceleration periods between the two scenarios.

The distance traveled during the non-deceleration periods are also equal between the two scenarios. However, in scenario 1 this distance is traversed at the initial velocity ($v_0$) and in scenario 3, this distance is traversed at the lower entry velocity ($v_E$). Since it takes longer for a vehicle to traverse a distance at a lower velocity, we can discern that the time to traverse the non-deceleration distance will be longer in scenario 3 than in scenario 1. Therefore, Scenario 3 represents a worse case scenario than Scenario 1.

We now compare scenario 2 and scenario 3 to determine which scenario requires the longer time for the driver to traverse the braking distance. While it is possible to algebraically compare the time calculations in the two scenarios, this would require advanced algebraic analysis beyond the scope of this paper. A simpler method is to use sample calculations.

Suppose a driver’s initial velocity is 40 mph (~59 fps) and their intersection entry velocity is 20 mph (~29 fps). Assume a maximum safe and comfortable deceleration, $a_{\text{max}}$, of 10 fps². The braking distance for an initial velocity of 59 fps is given by:

$$x_{Br} = \frac{v_0^2}{2a} = \frac{59^2}{20} = \frac{3481}{20} = 174 \text{ ft} \quad (18)$$

The time to traverse this braking distance while decelerating from 59 fps to 29 fps is given by:

$$T_{Br} = \frac{x_{Br}}{v_{av}} = \frac{174}{44} = 3.96 \text{ sec} \quad (19)$$

Therefore, the time to traverse the braking distance assuming the driver-vehicle motion in Scenario 2 is 3.96 seconds.
The time to traverse the braking distance in Scenario 3 is given by:

\[ T_{Br} = \frac{(v_0 - v_E)}{a} + \frac{v_E}{2a} = \frac{(59 - 29)}{10} + \frac{29}{20} = 4.45 \]  

(20)

The table below compares the minimum time to traverse the braking distance for scenarios 2 and 3 for various approach and entry speeds.

<table>
<thead>
<tr>
<th>V_0 (ft/sec)</th>
<th>V_E (ft/sec)</th>
<th>V_0 (ft/sec)</th>
<th>V_E (ft/sec)</th>
<th>V_0 (ft/sec)</th>
<th>V_E (ft/sec)</th>
<th>V_0 (ft/sec)</th>
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<th>V_0 (ft/sec)</th>
<th>V_E (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>20</td>
<td>45</td>
<td>15</td>
<td>45</td>
<td>12</td>
<td>45</td>
<td>15</td>
<td>45</td>
<td>12</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>4.57</td>
<td>4.95</td>
<td>5.21</td>
<td>4.51</td>
<td>4.77</td>
<td>4.99</td>
<td>3.67</td>
<td>4.03</td>
<td>4.25</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>5.13</td>
<td>5.50</td>
<td>5.72</td>
<td>4.40</td>
<td>4.77</td>
<td>4.99</td>
<td>3.67</td>
<td>4.03</td>
<td>4.25</td>
</tr>
</tbody>
</table>

As can be seen, Scenario 3 always yields the longest time to traverse the braking distance regardless of the initial and entry velocities and therefore is the model which a driver requires the longest time traverse the minimum stopping distance. Consequently, Scenario 3 must be the boundary condition.

The yellow change interval calculated using the Scenario 3 model provides sufficient yellow time for the other models of reasonable driver-vehicle motion we have considered. Consequently, adopting a protocol based on any other model would not provide a sufficient yellow interval because some subset of reasonable drivers would periodically encounter a dilemma zone and be placed in physical and legal jeopardy, as the red light may illuminate prior to them crossing the limit line.

**The Extended Solution - A Recommended Protocol for Turning Lanes**

Based on the above discussion, the following recommended protocol should be adopted for calculating the minimum yellow change interval for turning maneuvers. This solution is an extension of the current protocol used to calculate the minimum yellow change interval for through movements. This Extended Kinematic Equation eliminates the dilemma zone currently being encountered by many motorists executing a turning maneuver on approach to a signalized intersection and allows the majority of all driver-vehicle combinations legally traveling on the roadway sufficient yellow time to traverse the minimum stopping distance prior to the light turning red:

\[ Y_{min} = t_{PR} + \frac{(v_0 - v_E)}{a_{max}} + \frac{v_E}{2a_{max}} \]  

(21)

Where:

- \( Y_{min} \) = Minimum yellow change interval (sec)
- \( t_{PR} \) = Maximum allocated driver-vehicle perception-reaction time (sec)
- \( v_0 \) = Maximum uniform initial/approach velocity (ft/sec)
- \( v_E \) = Maximum intersection entry velocity (ft/sec)
- \( a_{max} \) = Maximum uniform safe and comfortable deceleration (fps\(^2\))

Algebraic simplification of Equation (21) yields:
The validity of the Extended Kinematic Equation is established in the following manner:

When \( v_E = v_0 \) (constant velocity), the protocol yields the ITE Kinematic Equation applicable for through movements.

When \( v_E = 0 \) (zero end velocity), the protocol yields the equation to calculate the minimum time to come to a complete stop:

\[
\begin{align*}
Y_{\text{min}} &= t_{PR} + \frac{v_0 - 1/2v_E}{a_{\text{max}}} \\
t_{\text{Stop}} &= t_{PR} + \frac{v_0}{a_{\text{max}}}
\end{align*}
\]

(22)

(23)

General Assumptions

All the models of driver-vehicle motion presented above, including those describing through movements, incorporate the following presumptions:

1. The vehicle is traveling in free-flow conditions (unimpeded, not within a queue, etc.).
2. The yellow indication illuminates at the moment the vehicle arrives at the minimum stopping distance.
3. When the yellow indication illuminates, the vehicle’s approach velocity is no greater than the 85th percentile speed or the posted speed limit, whichever is higher.

Assumed Initial Approach Speed

The proper approach speed for calculating the minimum yellow change interval is the 85th percentile approach speed as determined under free-flow conditions as determined by a speed study. However, the value of approach speed should not be less than the speed limit. This is consistent with the current proposed recommended practice which states that for through movements, the value used for the approach speed “should not be less than the posted speed limit”.

Engineers recognize the inherent contradiction and fundamental flaw in adopting a protocol that uses an approach velocity less than the legal speed limit. This recognition should extend to vehicles in turning lanes as well, regardless of whether some portion of vehicles are traveling below this legal limit.

While it is true that the length and configuration of many turning lanes may require a driver to decelerate prior to entering the lane, this is not true for numerous lane configurations routinely encountered by motorists on the nation’s roadways. For example, drivers in a through lane that drops into a turning lane as well as drivers approaching in long turning lanes do not need to slow down until they are closer to the intersection than the minimum braking distance.

Further, adopting a protocol that allows for a minimum yellow change interval to be based on an approach velocity less than the legal speed limit violates sound engineering practices as well as one of the basic intents of the protocols to yield reasonable yellow change intervals that enhance intersection safety and provide for the legal movement of vehicles and pedestrians.

Yellow change intervals set using an approach speed less than the posted speed limit is inherently dangerous and unfair. Some portion of drivers confronting these shorter yellow intervals, who are
otherwise behaving reasonably and legally by traveling at the posted speed limit, will inevitably encounter a dilemma zone and be forced to run the red light or brake at a higher than expected rate. This puts those drivers and other roadway users at risk of a collision. It also may subject them to inequitable enforcement.

**Assumed Intersection Entry Speed**

Assumed intersection entry speeds should be determined using engineering judgement. Generally, drivers entering an intersection to conduct a left turn, do so at approximately 20 mph depending on the radius of the curve they must negotiate. Right-turning drivers generally negotiate the turn at approximately 12 mph.

However, an entry speed can be calculated based on the *curve design speed* (aka “maximum safe speed” or “advisory speed”) which has been published by the Institute of Transportation Engineers.

On roadways with no banking, the equation to determine the curve design speed reduces to:

\[ v_{cds} = \sqrt{15 \times R \times f} \]  

(24)

Where:

- \( v_{cds} \) = Curve design speed (mph)
- \( R \) = Curve Radius (ft)
- \( f \) = Side friction factor; for speeds of 20 mph or less, \( f = 0.28 \)

The above equation provides the velocity in mph. To convert to fps, multiply by 1.467. For speeds of 20 mph or less, simplify the equation by substituting 0.28 for \( f \):

\[ v_{cds} = 1.467 \times \sqrt{15 \times 0.28 \times R} = 1.467^2 \times \sqrt{15 \times 0.28 \times R^2} \]

\[ = \sqrt{1.467^2 \times 4.2 \times R^2} = \sqrt{2.5 \times 4.2} = \sqrt{9.03} \]

(25)

Where the point of curvature is at the limit line, the curve design speed (turning velocity) is assumed to be the speed at which the vehicle crosses the limit line. Where the point of curvature is further forward from the limit line, we can either assume that the vehicle reaches its turning velocity at the limit line and then maintains a constant velocity into the turn or that the vehicle crosses the limit line at some higher speed and then continues to decelerate to achieve the turning velocity.

For the second assumption, we can calculate the speed of the vehicle at the limit line in the following manner:

\[ v_i^2 = v_f^2 - 2ad \]

(26)

Where:

- \( v_i \) = The velocity at the limit line
- \( v_f \) = The turning velocity
- \( d \) = The distance from the limit line to the point of curvature (turning location)
- \( a_{max} \) = The deceleration rate \( \leq -10 \text{ fps}^2 \)
Solving for $v_i$ by taking the square root gives:

$$v_i = \sqrt{v_f^2 - 2ad}$$  \hspace{1cm} (27)

and since:

$$v_{cds} = v_f = \sqrt{9.03R}$$  \hspace{1cm} (28)

then substituting $\sqrt{9.03R}$ for $v_f$ gives:

$$v_i = \sqrt{9.03R - 2ad}$$  \hspace{1cm} (29)

Note that in this situation the deceleration is negative so the two terms within the square root function are added together.

Note also that where the point of curvature is at the limit line, $d = 0$, and where the vehicle maintains a constant speed into the intersection, $a = 0$. In both cases, $v_i$ is simply the curve design speed calculated using $v_{cds} = \sqrt{9.03R}$

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